Analysis of channel and frequency offset estimator for downlink of mimo-ofdm system

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Abstract: The issues of frequency offset and channel estimation are considered for the downlink of multiple input multiple-output orthogonal frequency-division multiplexing systems This difficulty is estimated using multiple CFOs comes from performance of communication in MIMO ODFM downlink receiver, since many factors are involved in these processes. A parallel interference cancelation strategy which significantly reduces computational complexity and compensate for CFO effect is adapted. The CRLB and mean square error performance of the estimator is also derived and verified through computer simulations. The Parallel interference cancelation channel and carrier frequency offset (PCFE) algorithm is established. And then comparing the results with conventional methods the factors of performance are analyzed.

Keywords: Channel estimation, frequency offset, multiple-input multiple-output orthogonal frequency-division multiplexing (MIMO-OFDM).

I. Introduction

OFDM is the most promising physical layer technology for high data rate wireless communication. In orthogonal frequency-division multiplexing (OFDM), the entire channel is divided into many narrow-band sub channels, which are transmitted in parallel to maintain high-data-rate transmission and, at the same time, to increase the symbol duration to combat ISI. The capacity of a wireless system can significantly be improved if multiple transmit and receive antennas are used to form multiple-input–multiple-output (MIMO) channels. It is proved in that, a MIMO system can improve the capacity by a factor of the minimum number of transmit and receive antennas for flat fading or narrow-band channels. For wideband transmission, it is natural to combine OFDM with MIMO.

Orthogonal frequency-division multiplexing (OFDM) systems have resistance against frequencyselective channel fading, and the multiple-input–multiple-output (MIMO) technique provides additional spatial diversity/multiplexing gain. Combining these two techniques provides both advantages and has attracted great attention. Channel estimation is a critical component in MIMO-OFDM communication systems. In addition to channel estimation, frequency offset estimation is another well-known crucial Issue in the performance of OFDM systems, which is even more critical in MIMO-OFDM systems.

When examining a cellular system, a mobile subscriber (MS) travelling from cell center to cell boundary faces varying service data rates due to inter cell interference. In multicellular concept all MIMO base stations shares data and channel impulse response (CIR) information among neighboring BSs However, each BS has its own independent local oscillator (LO) with a relevant carrier frequency offset (CFO) to the receiving MS. The CFO induces a phase error that causes a loss of orthogonality among sub-carriers and, therefore, increases inter-cell and inter-carrier interference. In the coordinated OFDM downlink, the BSs utilize all the subcarriers. Therefore, it is not obvious how CFO can be applied to the coordinated OFDM downlink.

The rest of paper is organized as follows: Section II gives an overview of the Relevant Work. Point to point MIMO-OFDM system and system model which we have studied in section III. Section IV discussed on performance analysis on proposed system. Section V discusses the simulation results describing the estimation of channel and frequency offsets. Our conclusion based on literature and future works are listed in section VI.

II. Relevant Work

This section provides an overview of previous studies that have been conducted by researchers in the field of MIMO-OFDM Wireless communication.

A Joint Channel and Frequency Offset Estimator for the Downlink of Coordinated MIMO-OFDM System[1] as a reference gives a scenario to coordinated systems, and propose a novel channel and frequency offset estimator that allows for simultaneous transmission. Forinter-carrierinterference mitigation, a closed-form estimate of frequency offsets and the channel is delivered based on the properties of the discrete Fourier transform and Schur- Hadamard product. For inter-cell interference elimination, a parallel interference cancelation (PIC) strategy is adopted to iteratively compensate for the CFO effect. Compared with conventional approaches, our method significantly reduces the computational complexity, because no matrix inversion

isutilized. Simulation results show that the proposed method not only maintains a fast convergent speed, but also remains both accurate and robust to variations in the frequency offsets. The difficulty in estimating multiple CFOs comes from the situation that the training sequences may lose their desired orthogonality during transmission and become correlated.

In [2], Ahmed *e al a*voided accounting for this correlation, and then estimated the CFOs independently. This avoidance is only proper when the discrete Fourier transform (DFT) size is small or the sequences are sufficiently short, which is not satisfied in practical systems.

In [3], B. Zarikoff and J. Cavers, consider the downlink of a single-carrier coordinated system, which is essentially a generalization on the MIMO beam former. A coordinated downlink must share transmit data and channel state information (CSI) within each group of BSs, and the CSI must be accurate in order for beam forming to be effective. Unfortunately, each BS has an independent local oscillator (LO) with its own carrier frequency offset (CFO). The CFOs can cause phase errors, which in turn degrade the performance of the beam former over the duration of a frame. Then they focus this paper on single-carrier transmission, and recognize that the effect of CFO in a multi carrier coordinated system is even more pronounced, with inter carrier inter ference in addition to the inter-user interference arising from beam forming degradation.

In [4], Besson and Stoica scheduled only one BS to transmit training signals at any symbol time, so as to avoid the correlation problem and reduce the estimation to a sequence of single CFO estimates. However, their approach requires more estimating intervals and lowers the received energy by the BS cardinality, compared with simultaneous transmission schemes. In contrast to the above two approaches, Zarikoff and Cavers [3] proposed a scheme for simultaneously estimating multiple frequency offsets in the downlink of coordinated MIMO systems. The CFO variables were updated by the Newton method through the convexity assumption of the log-likelihood function over CFOs, while the CIR parameters were acquired through least square (LS) estimator. However, this approach usually has high computational complexity, especially when DFT size and/or number of BSs is large.

In [5], Y.-J.Liang and J.-F. Chang adopted on MIMO spatial domain frequency offset compensators and applied it of the frequency offset estimation in both the spatial and temporal domains in MIMO-OFDM systems. In contrast to these existing methods based on iterative updates between offset estimation and channel estimation, our scheme is based on the properties of discrete Fourier transform (DFT) and Hadamard product, thus achieving iteration free and closed-form solution. An approximate MSE is derived, and its accuracy is verified through simulations. Analytical and simulation results show that channel estimation and frequency offset compensation could perform well in both time and spatial domains. As a consequence, our method could effectively mitigate the interference and perform better. The computational complexity is also analyzed and shown to be much lower than those of conventional iterative methods

Notation: Upper (lower) boldface letters are used for matrices(column vectors). $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and Hermitian operators, respectively. We use ©and \otimes todenote the element-wise (Schur-Hadamard) and Kroneckerproducts, respectively. Also, $0_{M \times N}$, $1_{M \times N}$ and I_N denote the $M \times N$ all-zeroes, $M \times N$ all-ones, and $N \times N$ identity matrices, respectively. Diag{a} denotes a diagonal matrix with a on the main diagonal. [A]*m*, *n* stands for the *m*th-row *n*th columnelement/submatrix of matrix A. tr{A} denotes the trace of a square matrix A, defined as the sum of the main diagonal elements of A.R(*a*) and Im(*a*) represent the real and imaginary parts of variable *a*, respectively. E[·] is used for the expectation operator and $j \triangleq \sqrt{-1}$.

III. System Modal

In cellular systems, the subscriber location affects the data throughput and decoding performance in downlink communications. As a MS travels from a cell center to its boundary, its receiving quality degrades. This phenomenon cannot be completely solved with power control; a condition of non-smooth and unequal-rate data transfer still exists and limits overall system performance. A multicellular system can bridge these differences and achieve a more uniform communication quality. In this work, we study the channel estimation of MIMO-OFDM systems with multiple frequency offsets in the downlink of a coordinated multicellular system.

3.1System Description

First, consider a coordinated MIMO-OFDM downlink system for transmitting independent data to a single MS. We recognize that the effect of CFOs to multiple receiving MSsis similar to the case of a single MS. Thus, the same estimation procedure for a single MS can be adapted to the other MSs. The coordinated transmitters consist of BBSs, each with *Nt*transmit antennas, while the receiving MS has Nr antennas. The (inverse) DFT size at the BSs and MS is equal to *K*. The training pilots are first OFDM modulated, and then transmitted through *Nt*antennas over a frequency-selective fading channel with *L* discrete-time taps for each transmit-receive pair. It is assumed that all transmitting antennas at a given BS share the same oscillator, as do

the receiving antennas at the MS. The specified MS has *B* CFOs (of $f0, \ldots, fB-1$ Hz), associated with the *B* involved BSs. For a subcarrier frequency of fFHz, the CFOs are normalized to the subcarrier frequency as vb=fb/fF, $b=0, \ldots, B-1$. We further assume that coarse frequency offset correction has been performed such that $vb \in [-1/2, 1/2]$, $\forall b = 0, \ldots, B-1$. The proposed frequency offset and channel estimation scheme is data-aided. The time-domain training symbols for the *n*th transmitting antenna of the *b*th BS at the *k*th sample of an OFDM symbol, including N_g cyclic prefix (CP) samples, are denoted as $\{sn, b[k], k = -Ng, \ldots, K-1\}$. Furthermore, we use [Sn, b]k, l= sn, b[k - l] for $k \in \{0, \ldots, K-1\}$ and $l \in \{0, \ldots, L-1\}$ to denote the *k*th-row and *l*th-columnelement of the training signal matrix for the *n*th transmitting antenna of the *b*th BS. Here one receiving antenna is considered in designing the joint frequency offset and channel estimator, since the same estimation procedure can be adopted on the other receiving antennas. However, the estimates from different receiving antennas of aBS share the same receiver LO. Hereinafter, we assume that perfect symbol synchronized reception is satisfied.

Let $hn, b \in CL$ be the channel vector associated with the *n*thtransmit antenna of the *b*th BS. The received vector **r** $\in CK$ after removing the CP is given by:

$$\mathbf{V}(v) = [\mathbf{V}_0(v_0) \cdots \mathbf{V}_{B-1}(v_{B-1})], (4)$$



Fig 1: The block diagram of MIMO-OFDM system

with vdenoting the frequency offset vector obtained by concatenating the CFOs of *B* BSs, i.e., $v=[v_{0,...},v_{B-1}]^T$, and $n \in \mathbb{C}^K$ represents a zero-mean noise vector that has a circularly symmetric complex Gaussian distribution with a covariance matrix of $\sigma_n^2 I_K$. In (3), $h_b = [h_{0,b}^H \dots h_{Nt-1,b}^H]^H$ is the CIR vector respective to the *b*th BS, with the sub-

vector for *n*th transmit antenna,
$$\mathbf{h}_{n,b}$$
. In (4), $\mathbf{V}_b(v_b) = \mathbf{d}_b \bigotimes \mathbf{1}_{1 \times NiL} \in \mathbf{C}_{K \times NiL}$, with $\mathbf{d}_b = \left[\mathbf{1}, e^{\frac{j2\pi vb}{K}}, \dots, e^{\frac{j2\pi vb}{K}}\right]^T$

3.2Training Sequences

In this work, we adopt the closed-form estimate of frequency offsets, proposed in our previous research [4], by designing the training sequences as cascaded orthogonal formats. That is, $S_b^H \mathbf{S}_b = \rho \mathbf{I}_{NtL}, \forall b = 0, \dots, B-1, (5)$

$$(S_b^m)^H S_b^m = \frac{\rho}{M} \mathbf{I}_{NtL}, \ \forall m = 0, \dots, M-1, (6)$$

Where ρ is a scalar, *m* and *M* are the sub-matrix index and the number of sub-matrices within \mathbf{S}_b , respectively, i.e., $\mathbf{S}_b = [(S_b^0)^H, \ldots, (S_b^{M-1})^H]^H$.

3.3 Point-to-Point MIMO-OFDM System

We first review the case of a point-to-point MIMO-OFDM system, i.e., B = 1 and the subscript b is then omitted in the following description of this subsection. Note that this point-to-point system is free of intercell interference. Thus the received vector **r** in (1) can be simplified as

$$r = \binom{r^0}{r^1} = \left(\binom{S^0}{S^1} \otimes \binom{V(v)^0}{V(v)^1} \right) h + \binom{n^0}{n^1}, \tag{7}$$

Where

$$V(v)^{1} = e^{j\pi v} . V(v)^{0}, \qquad (8)$$

Since $\mathbf{S}^{i}(i=0, 1)$ is orthogonal and $\frac{v}{K}$ is much smaller than one (v ranges in [-1/2, 1/2]), after left-multiplying the received vector \mathbf{r} by $\mathbf{\tilde{S}}^{H}$ and using Taylor series expansions, we have

$$\tilde{r}^{0} = (S^{0})^{H} r^{0} \cong \frac{\rho}{M} \left(I \ \mathbb{O}V^{o}_{(1:NtL)} \right) h + (S^{0})^{H} n^{0}, \quad (9)$$
$$\tilde{r}^{1} = (S^{1})^{H} r^{1} \cong \frac{\rho}{M} \left(I \ \mathbb{O}V^{1}_{(1:NtL)} \right) h + (S^{1})^{H} n^{1}, \quad (10)$$

Where $\mathbf{A}_{(1,N)}$ consists of the first N rows of A. Ignoring the noise terms in (9) and (10), the estimated frequency offset, \hat{v} , can be derived as [4]

$$\hat{v} = \frac{1}{\pi} \arg\left(\sum_{i=0}^{NtL-1} \frac{\tilde{r}^{1}[i]}{\tilde{r}^{0}[i]}\right), (11)$$

Where $\mathbf{r}[i]$ denotes the *i*th element of \mathbf{r} and arg (·) returns the phase of its argument.

Fig. 1 depicts the estimation procedure of our proposed algorithm for a coordinated MIMO-OFDM downlink system, where $\mathbf{c}_{n,b}$ denotes the pilot-tone vector on the *n*th transmitting antenna of the *b*th BS, W(*l*) =diag(1, $e^{j2\pi l/K}, \ldots, e^{j2\pi l/(K-1)/K}$), $l = 0, \ldots, L-1$, models the channel taps, and \mathbf{F}_K represents the K-point discreteFourier transform (DFT) matrix. The transmit matrix of the *b*th BS, \mathbf{S}_b , is concatenated as $\mathbf{S}_b = [\mathbf{s}_{0,b}, \ldots, \mathbf{s}_{Nt-1,b}]$, with $\mathbf{s}_{n,b} =$ $F_{K}^{-1}W(l)\mathbf{c}_{n,b}$ [5]. After the offset matrix deterioration \mathbf{V}_{b} , channel fading \mathbf{h}_{b} , and white Gaussian noise addition \mathbf{n} , the combined received vector \mathbf{r} is extracted along the $\mathbf{S}_{0,-}$, \mathbf{S}_{B-1} signal subspaces. By substituting the extracted signal vectors into Algorithm 1, we can acquire the estimated CIR, \hat{h} , and frequency offsets, $\hat{v}_0, \dots, \hat{v}_{B-1}$.

Before stepping into the next section, we give some remarks for the proposed algorithm:

- Since \mathbf{r} is modeled by a Hadamard (element-wise) product as in (1), it is intuitive to estimate the frequency offsets through element-wise divisions based on the well-designed "cascaded orthogonal" pilots.
- 2) For the elimination of inter-cell interference, since the off-diagonal entries of the matrix $\tilde{S}_{(v)}$ are much smallerthan the main diagonal elements, its inverse could be wellapproximated by the Taylor series expansion.

IV. **Performance Analysis**

A.Cramér-Rao Lower Bound

The Cramer-Rao bound establishes the lower limit on how much "information" about an unknown probability distribution parameter a set of measurements carries. More specifically, the inequality establishes the minimum variance for an unbiased estimator of the underlying parameter, θ , of approbability distribution, $p(x; \theta)$. Three important points must be kept in mind about the Cramer-Rao bound: 1) the bound pertains only to unbiased estimators, biased estimators may violate the lower bound 2) the bound may be unreachable in practice, and 3) maximum likelihood estimators achieve the lower bound as the size of the measurement set tends to infinity. The following note outlines the proof of the Cramer-Rao bound for a single parameter Before proving the Cramer-Rao bound, let's first establish several key components of the proof. The score, S is defined as

$$S = \frac{\partial}{\partial \theta} \log p(x, \theta)$$
(12)

 $\partial p(x,\theta)$ $\partial \theta \quad p(x,\theta)$

The expected value of the score, $E{S}$, is

$$E\{S\} = \int_{-\infty}^{\infty} \left(\frac{\partial p(x,\theta)}{\partial \theta} \frac{1}{p(x,\theta)}\right) p(x,\theta) dx ,$$

= 0, (13)

The variance of the score, $Var{S}$, is termed the Fisher information (denoted $I(\theta)$). The score for a set of Nindependent identically-distributed (i.i.d.) variables is the sum of the respective scores,

$$s(x_1, x_2, \dots, x_N) = \frac{\partial}{\partial \theta} \log p(x_1, x_2, \dots, x_N), \quad (14)$$
$$= \sum_{i=1}^{n} \frac{\partial}{\partial \theta} \log p(x, \theta), \quad (15)$$
$$= \sum_{i=1}^{N} S(xi), \quad (16)$$

Similarly, it can be shown that the Fisher information for the set is $NI(\theta)$.

Theorem: The mean square error of an unbiased estimator, g, of a probability distribution parameter, θ , is lower bounded by the reciprocal of the Fisher information, I(θ), formally,

$$Var(g) \ge \frac{1}{I(\theta)},$$
 (17)

This lower bound is known as the Cramer-Rao bound.

Proof: Without loss of generality, the proof considers the bound related to a single measurement. The general case of a set of i.i.d. measurements follows in a straightforward manner. By the Cauchy-Schwarz inequality,

$$(E\{(S - E\{S\})(g - E\{g\})\})^2 \le E\{(S - E\{S\})^2\}E\{(g - E\{g\})^2\}$$
(19)

Further expansion of (19) yields,

$$\left(E\{Sg - E\{S\}g - SE\{g\} + E\{S\}E\{g\}\}\right)^2 \le E\{S^2 - 2SE\{S\} + E\{S\}^2\} Var\{g\},\tag{18}$$

Since the expected value of the score is zero, $E\{S\} = 0$, (11) simplifies as follows, $(E\{Sg - SE\{g\}\})^2 \le E\{S\}^2 Var\{g\},$ (19)

$$(E\{Sg\} - E\{SE\{g\}\})^{2} \leq I(\theta)Var\{g\},$$

$$(E\{Sg\} - E\{S\}E\{g\})^{2} \leq I(\theta)Var\{g\},$$

$$(E\{Sg\})^{2} \leq I(\theta)Var\{g\},$$
(20)

$$\left(\int_{-\infty}^{\infty} \left(\frac{\partial p(x,\theta)}{\partial \theta} \frac{1}{p(x,\theta)}\right) g(x) p(x,\theta) dx\right)^{2} \leq I(\theta) Var\{g\},(21)$$

$$\left(\frac{\partial}{\partial \theta} \int_{-\infty}^{\infty} g(x) p(x,\theta) dx\right)^{2} \leq I(\theta) Var\{g\},(22)$$

$$\left(\frac{\partial}{\partial \theta} E\{g\}\right)^{2} \leq I(\theta) Var\{g\},$$
(23)

Since g is an unbiased estimator (i.e., $E\{g\} = \theta$), (27) becomes,

$$\left(\frac{\partial}{\partial\theta}\theta\right)^2 \le I(\theta) Var\{g\},\tag{24}$$

$$1 \le I(\theta) Var\{g\},\tag{25}$$

Thus,

$$Var\{g\} \ge \frac{1}{I(\theta)},\tag{26}$$

B. Mean Square Error

In order to immediately access the performance of our algorithm, we derive the mean squared error of the estimated channel vector after the iterative CFO compensations and channel estimation. In the following derivation, without loss of generality, we assume that the channel vectors of different transmit and receive antenna pairs are statistically independent and have the same autocorrelation matrix, i.e., $C_h = I_{BNt} \otimes C_{h0}$, where the autocorrelation matrix of each channel is $C_{h0} = \text{diag}\{\sigma_0^2, \ldots, \sigma_{L-1}^2\}$. Now, the MSE of the estimated channel vector can be derived as:

$$MSE \triangleq \frac{E[(h-h)^{n}(h-h)]}{BN_{t}L},$$

$$\geq \frac{\sigma_{n}^{2}}{\rho} \left(1 + \frac{4\pi^{2}K(B-1)}{3BN_{t}L} \sum_{i=0}^{B-1} \xi_{i}^{2} \right), \qquad (27)$$

$$= \frac{BN_t}{K\gamma} + \frac{4\pi^2(B-1)}{3L\gamma} \sum_{i=0}^{B-1} \xi_i^2, \qquad (28)$$

Where

$$\hat{h} = \left(S^{H}\left(S \otimes V(v)\right)\right)^{-1} S^{H} r, \qquad (29)$$

And

$$\gamma \triangleq BN_t \rho / (K\sigma_n^2), \tag{30}$$

Defines the averaged signal-to-noise ratio(SNR) at the received antenna and $\xi_i = v_i - \hat{v}_i$ is the ith remaining frequency error after frequency offset compensation. Setting $\xi = [\xi_0, ..., \xi_{B-1}]^T = 0_{B \times 1}$ the lower bound obtain from (30) as

$$MSE \ge MSE|_{\xi=0} = \frac{BN_t}{K\gamma},\tag{31}$$

The lower bound in (31) increases with increase in B and N_t and decreases with increase in K. This phenomenon is proportional to cardinalities of transmitter antennas and base stations.

V. Vsimulation Results

In this paper, we propose to evaluate the performance of LS and LMMSE estimation techniques for Downlink systems. The cyclic prefix inserted at the beginning of each OFDM symbol is usually equal to or longer than the channel length in order to suppress ICI and ISI. However, the CP length can be shorter than the channel length because of some unforeseen behavior of the channel. Simulation results show that in the case where the CP length is equal to or longer than the channel length, the LMMSE performs better than LS estimator but at the cost of the complexity because it depends on the channel and noise statistics. In the other case, LMMSE provides better performance only for low SNR values and begins to lose its performance for higher SNR values. In other hand, LS shows better performance than LMMSE in this range of SNR values.

The LS estimator is shown to be the basic algorithm and gives regular results used with practically all the schemes of channel estimation, the LS estimator will be expressed as a ratio between the input data sequence and the output. The LMS estimator uses one tap LMS adaptive filter at each frequency. The first value is found directly through LS and the following values are calculated based on the previous estimation and the current channel output. The LMS estimator is used mainly for the tracking of the channel and is usually clustered with an equalizer or a decision feedback equalizer. The LMMSE estimator is of considerable complexity, since a matrix inversion is needed every time the training data in exchanges. The Fig 2 gives the plot of different channel estimators typically used in MIMO OFDM systems.



Fig 2: Channel Estimator in MIMO-OFDM system.

In Fig 3 the legend TD LMMSE refer to a time domain channel estimation technique, TDD LMMSE refer to the technique in which the channel covariance is ignored in the estimation and TD Qabs LMMSE refers to estimation in which the smoothing matrix is involved. LMMSE techniques are complex and computationally intensive TDD LMMSE and TD Qabs LMMSE reduces the complexity and computational time.



Fig 3: A time domain channel estimation in MIMO-OFDM system



Figure 4 shows CFO estimation by using three different techniques, once by using the phase difference between CP and the corresponding rear part of an OFDM symbol, once by using LS technique in using the phase difference between two repetitive preambles, and once by using MMSE technique the phase difference between pilot tones in two consecutive OFDM symbols. the mean squared CFO estimation errors decrease as the SNR of received signal increases Performances of estimation techniques vary depending on the number of samples in CP, the number of samples in preamble, and the number of pilot tones, used for CFO estimation.

VI. Conclusion

The results presented in this paper suggest that the channel and frequency offset is analytically predictable for downlink of MIMO OFDM system. The aim of the iterative PIC approach is to eliminate the inter-cell interference, whereas the aim of the joint channel estimator and frequency offset compensator is to mitigate the inter-carrier interference [1]. The CRLB and mean square error performance of the estimator is also derived and verified through computer simulations. This work could be extended in future to determine dependent factors that contribute to Multicellular communication. This would help in the accurate prediction of channel and frequency offset. The future work can also focus less complex estimator approach and its robustness to variations of CFOs

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